ECO227Y5 Tutorial 1

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Contact Information and Logistics

- Email: William.Hsu@mail.utoronto.ca
- Open email policy, do not hesitate to ask any questions
- I will try to respond within 48 hours

Recommended Revisions/Preparations

- First Year Calculus: Limits, Series and Sums, Derivatives and Optimization, Integrals (Single Variable)
- Upper Year Calculus: Partial Derivatives, Double Integrals
- Feel free to ask about these concepts in my office hours or email

Succeeding in ECO227

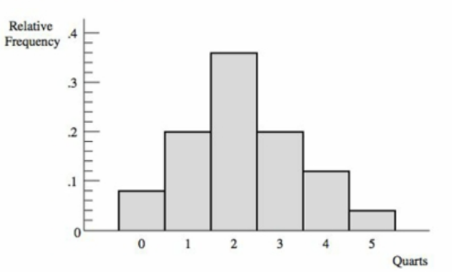
- Do the homework prior to coming to tutorial
- Ask questions if any confusions arise
- Try to reprove theorems discussed in class
- Understand *why* instead of *how* when completing questions

Office Hours

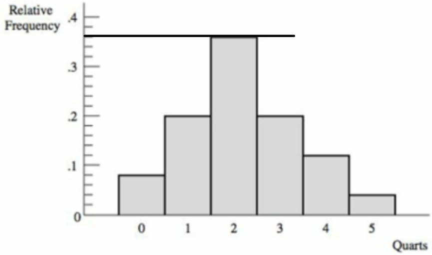
- Time: Mondays, 2:00 - 3:00
- Location: Economics Teaching Assistant Aid Centre, KN 3278
- To get to the room, take the elevator in Kaneff to the third floor
- I can host extra office hours closer to the tests

Question 1.6

Question: The relative frequency histogram given next was constructed from data obtained from a random sample of 25 families. Each was asked the number of quarts of milk that had been purchased the previous week.



Question 1.6

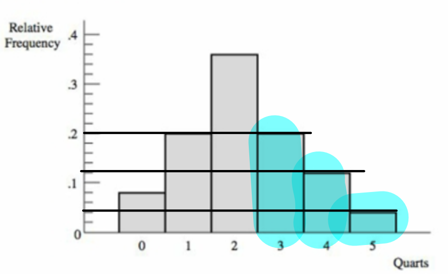


a) Use this relative frequency histogram to determine the number of quarts of milk purchased by the largest proportion of the 25 families. The category associated with the largest relative frequency is called the modal category.

The largest proportion = The group with the highest relative frequency.

We see that 2 quartz has the highest relative frequency, at 0.36 out of 1.00

Question 1.6



b) What proportion of the 25 families purchased more than 2 quarts of milk?

We want the combined relative frequencies of all quartz > 2.

We observe that 3 quartz has a relative frequency of 0.20, 4 quartz has 0.12, and 5 quartz has 0.04.

Thus, combined relative frequency = 0.20 + 0.12 + 0.04 = 0.36

Question: Is there another way to get the answer without adding all the r.f. ?

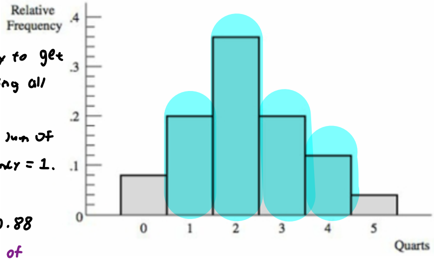
Answer: Yes! we know the sum of all relative frequency = 1.

Thus if we do

$1 - 0.08 - 0.04 = 0.88$

r.f. of 0 quartz

r.f. of 5 quartz



lower bound

upper bound

c) What proportion purchased more than 0 but fewer than 5 quarts?

We want the combined relative frequency of $0 < \#\text{quartz} < 5 \Rightarrow \#\text{quartz} = 1, 2, 3, 4.$

1 quartz: 0.20

2 quartz: 0.36

3 quartz: 0.20

4 quartz: 0.12

Thus, combined relative frequency = $0.20 + 0.36 + 0.20 + 0.12 = 0.88$

Question 1.11

Question: The following results on summations will help us in calculating the sample variance s^2. Use the following to show that:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right].$$

Let c be a real number, the following hold:

i.) $\sum_{i=1}^{n} c = nc$ Ex. $\sum_{i=1}^{5} 2 = 2+2+2+2+2 = 2(1+1+1+1+1) = 2(5) = 10$

ii.) $\sum_{i=1}^{n} cy_i = c\sum_{i=1}^{n} y_i$ Ex. $\sum_{i=1}^{3} 2i = 2(1) + 2(2) + 2(3) = 2(1+2+3) = 2\sum_{i=1}^{3} i$

iii.) $\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$ Ex. $\sum_{i=1}^{3} i + i^2 = 1 + 1^2 + 2 + 2^2 + 3 + 3^2$

$= (1+2+3) + (1^2 + 2^2 + 3^2)$

$= \sum_{i=1}^{3} i + \sum_{i=1}^{3} i^2$

Question 1.11

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Notice: $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

Proof: Let $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

↓ Expansion

$$= \frac{1}{n-1}\sum_{i=1}^{n} y_i^2 - 2y_i\bar{y} + \bar{y}^2.$$

↓ using rule iii

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} 2\bar{y}y_i + \sum_{i=1}^{n}\bar{y}^2\right)$$

↓ rule ii as $\bar{y}$ is a constant

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i + n\bar{y}^2\right)$$

↓ Defn of $\bar{y}$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - 2\left(\frac{1}{n}\sum_{n=1}^{n} y_i\right)\sum_{i=1}^{n} y_i + n\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^2\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{2}{n}\left(\sum_{i=1}^{n} y_i\right)^2 + \frac{n}{n^2}\left(\sum_{i=1}^{n} y_i\right)^2\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i - \frac{2}{n}\left(\sum_{i=1}^{n} y_i\right)^2 + \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right]$$

↓ combine like terms

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right] \blacksquare$$

Question 1.22

Question: Prove that the sum of the deviations of a set of measurements about their mean is equal to zero; that is,

$$\sum_{i=1}^{n}(y_i - \bar{y}) = 0.$$

Visually:

consider deviations from the left of the mean as negative

consider deviations from the right of the mean as positive

Proof: Consider $\sum_{i=1}^{n}(y_i - \bar{y})$.

By rule iii)

$$\sum_{i=1}^{n}(y_i - \bar{y}) = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \bar{y}$$

Recall: $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$$\Rightarrow \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \bar{y} = \sum_{i=1}^{n} y_i - n\bar{y}$$

$$\Rightarrow \sum_{i=1}^{n} y_i - n\bar{y} = \sum_{i=1}^{n} y_i - n\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} y_i = 0$$ as needed. ■

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Question 1.24

Question: Aqua running has been suggested as a method of cardiovascular conditioning for injured athletes and others who desire a low-impact aerobics program. In a study to investigate the relationship between exercise cadence and heart rate,1 the heart rates of 20 healthy volunteers were measured at a cadence of 48 cycles per minute (a cycle consisted of two steps). The data are as follows:

87	109	79	80	96	95	90	92	96	98
101	91	78	112	94	98	94	107	81	96

Question 1.24

87	109	79	80	96	95	90	92	96	98
101	91	78	112	94	98	94	107	81	96

a) Use the range of the measurements to obtain an estimate of the standard deviation.

Range = max(x) − min(x)

max(x) = 112

min(x) = 78

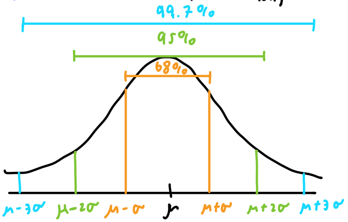
Range = 112 − 78 = 34

Thus, we can assume that the range $= (\mu + 3\sigma) - (\mu - 3\sigma) = 6\sigma$

$$\text{Range} = 6\sigma \Rightarrow \sigma = \frac{\text{Range}}{6} = \frac{34}{6} \simeq 5.67$$

Assumption: let the underlying distribution be normal. Thus, we may use the Empirical Rule.

Empirical Rule: Given a normally distribution, we have,



Almost all data fall within 3 standard deviations from the mean.

87	109	79	80	96	95	90	92	96	98
101	91	78	112	94	98	94	107	81	96

b) Construct a frequency histogram for the data. Use the histogram to obtain a visual approximation to $\bar{y}$ and s.

Bins: $75 \le x < 80$

$[75, 80)$: 2

$[80, 85)$: 2

$[85, 90)$: 1

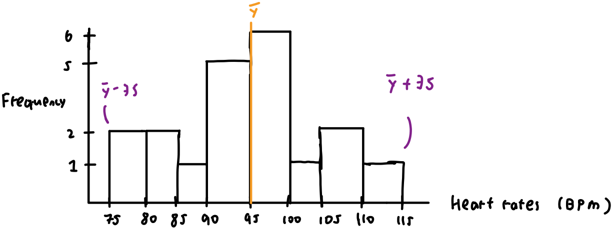
$[90, 95)$: 5

$[95, 100)$: 6

$[100, 105)$: 1

$[105, 110)$: 2

$[110, 115)$: 1



$\bar{y} \simeq 95$

Question 1.24

87	109	79	80	96	95	90	92	96	98
101	91	78	112	94	98	94	107	81	96

c) Calculate $\bar{y}$ and s. Compare these results with the calculation checks provided by parts (a) and (b).

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ $\quad n=20$

$\bar{y} = \frac{1}{20}(87 + 109 + \cdots + 96) = 93.7$

$s = \sqrt{s^2}$ — Result from Question 1.12

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right]$

$s^2 = \frac{1}{19}\left[(87^2 + 109^2 + \cdots + 96^2) - \frac{1}{20}(87 + 109 + \cdots + 96)^2\right]$

$s^2 = \frac{1}{19}\left[\frac{8671}{5}\right] = 91.274 \Rightarrow s = \sqrt{91.274} = 9.55$

Question 1.24

87	109	79	80	96	95	90	92	96	98
101	91	78	112	94	98	94	107	81	96

d) Construct the intervals $\bar{y} \pm$ ks, k $=$ 1, 2, and 3, and count the number of measurements falling in each interval. Compare the fractions falling in the intervals with the fractions that you would expect according to the empirical rule.

$\bar{y} + s = 93.7 + 9.55 = 103.25$

$\bar{y} - s = 93.7 - 9.55 = 84.15$

$\bar{y} + 2s = 93.7 + 2(9.55) = 112.8$

$\bar{y} - 2s = 93.7 - 2(9.55) = 74.6$

$\bar{y} + 3s = 93.7 + 3(9.55) = 122.35$

$\bar{y} - 3s = 93.7 - 3(9.55) = 65.05$

$\bar{y} \pm s = [84.15, 103.25]$; $\frac{13}{20} = 65\%$ of data

$\bar{y} \pm 2s = [74.6, 112.8]$; $\frac{20}{20} = 100\%$ of data

$\bar{y} \pm 3s = [65.05, 122.35]$; $\frac{20}{20} = 100\%$ of data