ECO227Y5 Tutorial 22

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Question 10.54

Question: Do you believe that an exceptionally high percentage of the executives of large corporations are right-handed? Although 85% of the general public is right-handed, a survey of 300 chief executive officers of large corporations found that 96% were right-handed.

(a) Is this difference in percentages statistically significant? Test using $\alpha = 0.01$.

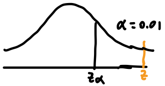
(b) Find the *p*-value for the test and explain what it means.

a.) $H_0: p_0 = 0.85$

$H_1: p_0 > 0.85$

$\hat{p} = 0.96$, $n = 300$ $\alpha = 0.01$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.96 - 0.85}{\sqrt{\frac{0.85 \cdot 0.15}{300}}} = 5.336$$



$z_\alpha = 2.33 < z$ so reject H_0 and accept H_1

so yes, different

b.) p value: $P(z > 5.34) \approx 0 < 0.01 = \alpha$

p value is essentially zero, if $p = 0.85$, the probability of observing a sample proportion of 96% or higher in a sample of 300 is virtually 0, so the H_0 is most likely false.

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Question 10.57

Question: A publisher of a newsmagazine has found through past experience that 60% of subscribers renew their subscriptions. In a random sample of 200 subscribers, 108 indicated that they planned to renew their subscriptions. What is the *p*-value associated with the test that the current rate of renewals differs from the rate previously experienced?

$H_0: p = 0.60$

$H_1: p \neq 0.60$

$\hat{p} = \frac{108}{200} = 0.54$

$n = 200$

$$z = \frac{0.54 - 0.60}{\sqrt{\frac{0.6\ 0.4}{200}}} = -1.732$$

$P(z < -1.73) = 0.0416$

but since this is a two sided test, we double probability

so p-value $= 0.0832$

Question 10.81

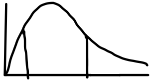
Question: From two normal populations with respective variances σ_1^2 and σ_2^2, we observe independent sample variances S_1^2 and S_2^2, with corresponding degrees of freedom $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$. We wish to test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$.

(a) Show that the rejection region given by

$$F > F_{\nu_1,\nu_2,\alpha/2} \quad \text{or} \quad F < \left(F_{\nu_2,\nu_1,\alpha/2}\right)^{-1},$$

where $F = S_1^2/S_2^2$, is the same as the rejection region given by

$$S_1^2/S_2^2 > F_{\nu_1,\nu_2,\alpha/2} \quad \text{or} \quad S_2^2/S_1^2 > F_{\nu_2,\nu_1,\alpha/2}.$$



a) $H_0 : \sigma_1^2 - \sigma_2^2 = 0$ - two sided from "$\neq$"

$H_2 : \sigma_1^2 - \sigma_2^2 \neq 0$

1. $F < \dfrac{1}{F_{v_2,v_1,\alpha/2}} \;\exists\; \dfrac{S_1^2}{S_2^2} < \dfrac{1}{F_{v_2,v_1,\alpha/2}} \;\exists\; \dfrac{S_2^2}{S_1^2} > F_{v_2,v_1,\alpha/2}$

2. $F > F_{v_1,v_2,\alpha/2} \;\exists\; \dfrac{S_1^2}{S_2^2} > F_{v_1,v_2,\alpha/2}$

can always get the right "greater than" value

Question: From two normal populations with respective variances σ_1^2 and σ_2^2, we observe independent sample variances S_1^2 and S_2^2, with corresponding degrees of freedom $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$. We wish to test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$.

(b) Let S_L^2 denote the larger of S_1^2 and S_2^2, and let S_S^2 denote the smaller. Let ν_L and ν_S denote the degrees of freedom associated with S_L^2 and S_S^2, respectively. Use part (a) to show that, under H_0,

$$P\left(S_L^2/S_S^2 > F_{\nu_L,\nu_S,\alpha/2}\right) = \alpha.$$

$H_0: \sigma_1^2 - \sigma_2^2 = 0$

$H_a: \sigma_1^2 - \sigma_2^2 = 0$

If $s_1^2 \geq s_2^2$ then $s_L^2 = s_1^2$, $v_L = v_1$, $v_S = v_2$

If $s_1^2 < s_2^2$ then $s_L^2 = s_2^2$, $v_L = v_2$, $v_S = v_1$

Let $F = \frac{s_L^2}{s_S^2}$

$$P\left(\frac{s_L^2}{s_S^2} > F_{v_L, v_S, \alpha/2}\right) = P\left(\frac{s_L^2}{s_S^2} > F_{v_1,v_2,\alpha/2}, s_1^2 \geq s_2^2\right) + P\left(\frac{s_L^2}{s_S^2} > F_{v_2,v_1,\alpha/2}, s_1^2 < s_2^2\right)$$

$$\Rightarrow P\left(\frac{s_L^2}{s_S^2} > F_{v_1,v_2,\alpha/2}\right) + P\left(\frac{s_L^2}{s_S^2} > F_{v_2,v_1,\alpha/2}\right)$$

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From part (a) $P\left(\frac{s_1^2}{s_2^2} > F_{v_1,v_2,\alpha/2} \vee \frac{s_2^2}{s_1^2} > F_{v_1,v_2,\alpha/2}\right) = \alpha = P\left(\frac{s_1^2}{s_2^2} > F_{v_1,v_2,\alpha/2}\right) + P\left(\frac{s_2^2}{s_1^2} > F_{v_2,v_1,\alpha/2}\right)$

Result: Just use the larger s^2 in the numerator and use the upper critical value only.

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Final Comments

- **Final Exam Office Hours**

  Saturday, April 4th · Time TBD

- **UEC Review Session** sign up on Instagram @UECUTM

  Tuesday, April 7th · 3:30–5:30 PM
  (I will be present from 3:30–4:30 PM)
  Location: DV 3140
  All 100-200 ECO courses

- **Connect with Me**

  LinkedIn: `https://www.linkedin.com/in/williamhanhsu/`

Thank you for coming to my tutorials and I wish you all the best in future endeavours!