ECO227Y5 Tutorial 2

William Hsu

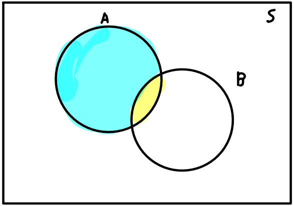
Department of Economics
University of Toronto Mississauga

September 22nd, 2025

Question 2.4

Question: If A and B are two sets, draw Venn diagrams to verify the following:

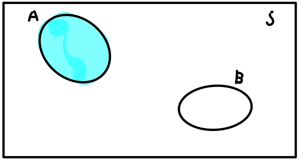
a. $A = (A \cap B) \cup (A \cap B^C)$



Let:

= $A \cap B$

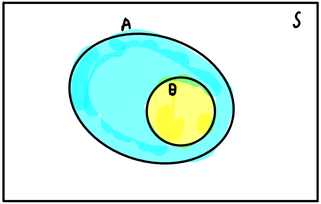
= $A \cap B^c$



Question 2.4

Question: If A and B are two sets, draw Venn diagrams to verify the following:
b) If B $\subset A$, then $A = B \cup (A \cap B^{c})$

Anywhere in S and not in B



let:

= B

= $A \cap B^c$

Question 2.10

Question: The proportions of blood phenotypes, A, B, AB, and O, in the population of all Caucasians in the United States are approximately .41, .10, .04, and .45, respectively. A single Caucasian is chosen at random from the population.
a) List the sample space for this experiment.
b) Make use of the information given above to assign probabilities to each of the simple events.
c) What is the probability that the person chosen at random has either type A or type AB blood?

a.) $S = \{A, B, AB, O\}$

- Given in question

b.) $P(A) = 0.41$, $P(B) = 0.10$, $P(AB) = 0.04$, $P(O) = 0.45$

c.) $P(A \cup AB) = P(A) + P(AB) - P(A \cap AB) = 0.41 + 0.04 + 0 = 0.45$

These events are mutually exclusive as you cannot have two blood types at the same time

Question 2.14

Question: A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

	Uses Eyeglasses for Reading	
Needs glasses	Yes	No
Yes	.44	.14
No	.02	.40

Question 2.14

Needs glasses	Uses Eyeglasses for Reading: Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

a) needs glasses. P(needs glasses) = 0.44 + 0.14 = 0.58

b) needs glasses but does not use them. P(needs glasses but does not use them) = 0.14

c) uses glasses whether the glasses are needed or not.

P(uses glasses) = 0.44 + 0.02 = 0.46

UNIVERSITY OF TORONTO

Question 2.15

Question: An oil prospecting firm hits oil or gas on 10 per cent of its drillings. If the firm drills two wells, the four possible simple events and three of their associated probabilities are as given in the accompanying table.

Simple Event	Outcome of First Drilling	Outcome of Second Drilling	Probability
E_1	Hit (oil or gas)	Hit (oil or gas)	.01
E_2	Hit	Miss	?
E_3	Miss	Hit	.09
E_4	Miss	Miss	.81

a) on the first drilling and miss on the second.
b) on at least one of the two drillings.

a.) Recall $\sum_{\text{All possible}} P(\text{simple events}) = 1$, thus $P(E_2) = 1 - P(E_1) - P(E_3) - P(E_4) = 1 - 0.01 - 0.09 - 0.81 = 0.09$

b.) $P(\text{At least one}) = 1 - P(\text{miss on both}) = 1 - P(E_4) = 1 - 0.81 = 0.19$

Alternatively, add up $P(E_1) + P(E_2) + P(E_3)$

UNIVERSITY OF TORONTO

Question 2.18

Question: Suppose two balanced coins are tossed and the upper faces are observed.
a) List the sample points for this experiment.
b) Assign a reasonable probability to each sample point. (Are the sample points equally likely?)

a.) $S = \{HH, HT, TH, TT\}$

b.) $P(HH) = P(HT) = P(TH) = P(TT) = 0.5 \cdot 0.5 = 0.25$

Question: Suppose two balanced coins are tossed and the upper faces are observed.
c) Let A denote the event that exactly one head is observed and B the event that at least one head is observed. List the sample points in both A and B.
d) From your answer to part (c), find P(A), P(B), $P(A \cap B)$, $P(A \cup B)$, and $P(A^C \cup B)$

c.) A = Exactly one head = {HT, TH}

B = At least one head = {HH, HT, TH}

d.) $P(A) = P(HT) + P(TH) = 0.25 + 0.25 = 0.50$

$P(B) = P(HH) + P(HT) + P(TH) = 0.25 + 0.25 + 0.25 = 0.75$

$A \cap B = A = \{HT, TH\}$

$P(A \cap B) = P(A) = 0.50$

$A \cup B = B = \{HH, HT, TH\}$

$P(A \cup B) = P(B) = 0.75$

$P(A^c \cup B) = P(\{HH, HT, TH, TT\}) = P(S) = 1$

$A^c = \{HH, TT\}$

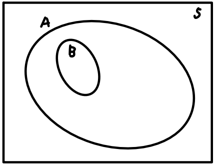
- This should be $B \subseteq A$

Question: If A and B are events and $B \subset A$, why is it obvious that
$$P(B) \leq P(A)?$$

Intuition: let $B \subseteq A$ both be events. Thus B has less or equal amounts of simple events compared to A.

By axiom 1, $P(\text{Any event}) \geq 0$. $P(B) = \sum P(\text{simple events in } B)$. $P(A) = \sum(\text{simple events in } A)$

But since B has less than or equal to number of simple events than A, $P(B) \leq P(A)$.



Proof: Let $B \subseteq A$.

mutually exclusive

$$P(A) = P[(A \cap B) \cup (A \cap B^c)] = P(A \cap B) + P(A \cap B^c).$$

$$P(B) = P((B \cap A) \cup (B \cap A^c)] = P(B \cap A) + P(B \cap A^c)$$

Notice that as $B \subseteq A$, $B \cap A^c = \phi$, $P(\phi) = 0$

So $P(B) = P(B \cap A) \leq P(A \cap B) + P(A \cap B^c) = P(A)$

As needed. ■

Question 2.28

Question: Four equally qualified people apply for two identical positions in a company. One and only one applicant is a member of a minority group. The positions are filled by choosing two of the applicants at random.
a) List the possible outcomes for this experiment.
b) Assign reasonable probabilities to the sample points.
c) Find the probability that the applicant from the minority group is selected for a position.

a.) Let m be the minority individual. Let a_1, a_2, a_3, denote the other individuals.

$S = \{a_1a_2, a_1a_3, a_2a_3, a_1m, a_2m, a_3m\}$ order does not matter as the positions are identical.

b.) $P(\text{any sample point}) = \frac{1}{6}$ Applicants are randomly selected

c.) $P(\text{minority individual selected}) = P(a_1m) + P(a_2m) + P(a_3m) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 0.5$

Question 2.29

Question: Two additional jurors are needed to complete a jury for a criminal trial. There are six prospective jurors, two women and four men. Two jurors are randomly selected from the six available.
a) Define the experiment and describe one sample point. Assume that you need describe only the two jurors chosen and not the order in which they were selected.
b) List the sample space associated with this experiment.
c) What is the probability that both of the jurors selected are women?

a.) The experiment involves selecting two random individuals for the same jury positions out of a group of six individuals, 2 women and 4 men.

b.) $S = \{m_1 m_2,\ m_1 m_3,\ m_1 m_4,\ m_1 w_1,\ m_1 w_2,\ m_2 m_3,\ m_2 m_4,\ m_2 w_1,\ m_2 w_2,\ m_3 m_4,\ m_3 w_1,\ m_3 w_2,\ m_4 w_1,\ m_4 w_2,\ w_1 w_2\}$

c.) $P(w_1 w_2) = \frac{1}{15}$

Question 2.71

Question: If two events, A and B, are such that P(A) = 0.5, P(B) = 0.3, and $P(A \cap B) = 0.1$, find the following:

a) $P(A|B)$
b) $P(B|A)$
c) $P(A|A \cup B)$
d) $P(A|A \cap B)$
e) $P(A \cap B|A \cup B)$

Recall: $P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

a.) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.3} = 0.\bar{3}$

b.) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.1}{0.5} = 0.2$

$* \; P(A \cup B) = 0.5 + 0.3 - 0.1 = 0.7$

c.) $P(A|A \cup B) = \dfrac{P(A \cup B|A) \cdot P(A)}{P(A \cup B)} = \dfrac{1 \cdot 0.5}{0.7} = \dfrac{5}{7}$

d.) $P(A|A \cap B) = \dfrac{P(A \cap (A \cap B))}{P(A \cap B)} = \dfrac{0.1}{0.1} = 1$

e.) $P(A \cap B|A \cup B) = \dfrac{P(A \cup B|A \cap B) \cdot P(A \cap B)}{P(A \cup B)} = \dfrac{1 \cdot 0.1}{0.7} = \dfrac{1}{7}$

UNIVERSITY OF TORONTO

Question 2.73

Question: Gregor Mendel was a monk who, in 1865, suggested a theory of inheritance based on the science of genetics. He identified heterozygous individuals for flower color that had two alleles (one r = recessive white color allele and one R = dominant red color allele). When these individuals were mated, 3/4 of the offspring were observed to have red flowers, and 1/4 had white flowers. The following table summarizes this mating; each parent gives one of its alleles to form the gene of the offspring.

	Parent 2	
Parent 1	r	R
r	rr	rR
R	Rr	RR

Question 2.73

Parent 1	Parent 2: r	R
r	rr	rR
R	Rr	RR

$P(rr) = P(Rr) = P(rR) = P(RR) = 0.25$

We assume that each parent is equally likely to give either of the two alleles and that, if either one or two of the alleles in a pair is dominant (R), the offspring will have red flowers. What is the probability that an offspring has

a) at least one dominant allele? $P(\text{At least one dominant Allele}) = P(rR) + P(Rr) + P(RR) = 0.75$

b) at least one recessive allele? $P(\text{At least one recessive Allele}) = P(rR) + P(Rr) + P(rr) = 0.75$

c) one recessive allele, given that the offspring has red flowers?

$$P(\text{one recessive Allele} \mid \text{red flowers}) = \frac{P(\text{one recessive Allele} \cap \text{red flowers})}{P(\text{red flowers})} = \frac{0.5}{0.75} = \frac{2}{3}$$

UNIVERSITY OF TORONTO

Question 2.76

Question: A survey of consumers in a particular community showed that 10 per cent were dissatisfied with plumbing jobs done in their homes. Half the complaints dealt with plumber A, who does 40 per cent of the plumbing jobs in the town. Find the probability that a consumer will obtain
a) an unsatisfactory plumbing job, given that the plumber was A.
b) a satisfactory plumbing job, given that the plumber was A.

a.) $P(\text{Unsatisfactory} \mid \text{plumber A}) = \dfrac{P(\text{Plumber A} \mid \text{unsatisfactory}) \cdot P(\text{unsatisfactory})}{P(\text{plumber A})} = \dfrac{0.5 \cdot 0.1}{0.4} = 0.125$

b.) $P(\text{Satisfactory} \mid \text{plumber A}) = 1 - P(\text{Unsatisfactory} \mid \text{plumber A}) = 1 - 0.125 = 0.875$

Question 2.77

Question: A study of the posttreatment behavior of a large number of drug abusers suggests that the likelihood of conviction within a two-year period after treatment may depend upon the offenders education. The proportions of the total number of cases falling in four education–conviction categories are shown in the following table:

	Status within 2 Years after Treatment		
Education	Convicted	Not Convicted	*Total*
10 years or more	.10	.30	.40
9 years or less	.27	.33	.60
Total	.37	.63	1.00

Suppose that a single offender is selected from the treatment program. Define the events:
A: The offender has 10 or more years of education.
B: The offender is convicted within two years after completion of treatment.

UNIVERSITY OF TORONTO

Question 2.77

	Status within 2 Years after Treatment		
Education	Convicted	Not Convicted	*Total*
10 years or more	.10	.30	.40
9 years or less	.27	.33	.60
Total	.37	.63	1.00

Suppose that a single offender is selected from the treatment program. Define the events:

A: The offender has 10 or more years of education.

B: The offender is convicted within two years after completion of treatment.

Find the following:

a) P(A) $= 0.40$

b) P(B) $= 0.37$

c) $P(A \cap B) = 0.10$

d) $P(A \cup B) = 0.40 + 0.37 - 0.10 = 0.67$

e) $P(A^C) = 1 - 0.40 = 0.60$

UNIVERSITY OF TORONTO

Question 2.77

Education	Status within 2 Years after Treatment: Convicted	Not Convicted	*Total*
10 years or more	.10	.30	.40
9 years or less	.27	.33	.60
Total	.37	.63	1.00

Suppose that a single offender is selected from the treatment program. Define the events:

A: The offender has 10 or more years of education.

B: The offender is convicted within two years after completion of treatment.

Find the following:

f) $P((A \cup B)^C)$ $= 0.33$

g) $P((A \cap B)^C)$ $= 0.90$

h) $P(A|B)$ $= \frac{0.10}{0.37} = 0.\overline{270}$

i) $P(B|A)$ $= \frac{0.10}{0.40} = 0.25$

Question 2.79

Question: If P(A) > 0, P(B) > 0, and P(A) < P($A|B$), show that
$$P(B) < P(B|A)$$

Proof: Let $P(A), P(B) > 0$. Let $P(A) < P(A|B)$

$$P(A) < P(A|B)$$

$$\Rightarrow P(A)P(B) < P(A \cap B)$$

$$\Rightarrow P(B) < \frac{P(A \cap B)}{P(A)}$$

As $P(A) > 0$, we can divide.

$$\Rightarrow P(B) < \frac{P(B|A) \cdot P(A)}{P(A)}$$

$$\Rightarrow P(B) < P(B|A) \quad \blacksquare$$

Question 2.80

Question: Suppose that $A \subset B$ and that P(A) > 0 and P(B) > 0. Are A and B independent? Prove your answer.

Claim: Independent if $B = S$, otherwise dependent.

Proof: let $A \subset B$, let $P(A), P(B) > 0$. We consider the two cases.

i.) $B \neq S$:

$P(B \mid A) = 1 \neq P(B)$ as $P(S) = 1$. Thus cannot be independent.

ii.) $B = S$:

$P(B \mid A) = 1 = P(B = S)$

$P(A \mid B) = P(A)$

$P(A \cap B) = P(A) \cdot P(B)$. Thus independent. ∎

UNIVERSITY OF TORONTO

Question 2.81

Question: Suppose that A and B are mutually exclusive events, with $P(A) > 0$ and $P(B) < 1$. Are A and B independent? Prove your answer.

Claim: If A and B are mutually exclusive, they cannot be independent, unless $P(B)=0$.

Proof: let A, B be mutually exclusive events. let $P(A)>0$, $P(B)<1$.

$P(A \cap B) = 0$ as $A \cap B = \phi$

If independent, we need $P(A \cap B) = P(A) \cdot P(B) = 0$

Since $P(A) > 0$, it is necessary that $P(B) = 0$ to be independent. ∎

Question 2.85

Question: If A and B are independent events, show that A and B^C are also independent. Are A^C and B^C independent?

i.) Proof: let A and B be independent events.

So $P(A \cap B) = P(A) \cdot P(B)$

$P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Consider $P(B^c | A) = 1 - P(B|A) = 1 - P(B) = P(B^c)$

Thus $P(B^c \cap A) = P(B^c) \cdot P(A)$ ∎

ii.) Claim: Yes, A^c and B^c are independent.

Proof: let A and B be independent.

we know A and B^c are independent, we know without loss of generality B and A^c are independent.

Consider
$$\begin{aligned} P(A^c \cap B^c) &= P(A^c | B^c) \cdot P(B^c) \\ &= (1 - P(A | B^c)) \cdot P(B^c) \\ &= (1 - P(A)) \cdot P(B^c) \\ &= P(A^c) \cdot P(B^c) \end{aligned}$$ ∎

Question 2.86

Suppose that A and B are events such that P(A) = 0.8 and P(B) = 0.7

a) Is it possible that $P(A \cap B) = 0.1$? Why or why not?

b) What is the smallest possible value for $P(A \cap B)$?

c) Is it possible that $P(A \cap B) = 0.77$? Why or why not?

d) What is the largest possible value for $P(A \cap B)$?

a.) Assume that $P(A \cap B) = 0.1$, Consider $P(A \cup B) = P(A) + P(B) - P(A \cap B) \in [0,1]$.

If $P(A \cap B) = 0.1 \Rightarrow P(A \cup B) = 0.8 + 0.7 - 0.1 = 1.4 \notin [0,1]$. So no!

b.) we need $P(A \cup B) \in [0,1]$, so the smallest value for $P(A \cap B)$ is when $P(A \cup B) = 1$.

$1 = 0.7 + 0.8 - P(A \cap B) \Rightarrow P(A \cap B) = 0.5$

c.) Assume $P(A \cap B) = 0.77$, Consider $P(A \cap B) = P(A|B) \cdot P(B) \Rightarrow 0.77 = P(A|B) \cdot 0.7 \Rightarrow P(A|B) = 1.1$

$P(A|B) = 1.1 \notin [0,1]$. So no!

d.) we need $P(A|B) \in [0,1]$. Notice that the largest $P(A \cap B)$ occurs if $P(A|B) = 1$.

$P(A \cap B) = 1 \cdot 0.7 = 0.7$

Question: why did I pick $P(A|B) \cdot P(B)$ instead of $P(B|A) \cdot P(A)$?

UNIVERSITY OF TORONTO

Question 2.96

If A and B are indpendent events with P(A) = 0.5 and P(B) = 0.2, find the following:

a) $P(A \cup B)$ $= P(A) + P(B) - P(A) \cdot P(B) = 0.5 + 0.2 - 0.5 \cdot 0.2 = 0.6$

b) $P(A^C \cap B^C)$ $= P(A^c | B^c) \cdot P(B^c) = P(A^c) \cdot P(B^c) = 0.5 \cdot 0.8 = 0.4$

c) $P(A^C \cup B^C)$ $= P(A^c) + P(B^c) - P(A^c \cap B^c) = 0.5 + 0.8 - 0.4 = 0.9$

Question 2.128

A plane is missing and is presumed to have equal probability of going down in any of three regions. If a plane is actually down in region i, let $1 - \alpha_i$ denote the probability that the plane will be found upon a search of the ith region, i = 1, 2, 3. What is the conditional probability that the plane is in

a) region 1, given that the search of region 1 was unsuccessful?
b) region 2, given that the search of region 1 was unsuccessful?
c) region 3, given that the search of region 1 was unsuccessful?

$$P(\underset{R_1}{\text{region } 1}) = P(\underset{R_2}{\text{region } 2}) = P(\underset{R_3}{\text{region } 3}) = \frac{1}{3}$$

$$P(\text{Found in region } i \mid \text{region } i) = 1 - \alpha_i, \quad i \in \{1, 2, 3\}$$

1 as the plane is not in region 1.

$$P(\underset{A}{\text{Not found in region } 1}) = \underbrace{P(A \mid R_1) \cdot P(R_1)}_{P(A \cap R_1)} + P(A \mid R_2) \cdot P(R_2) + P(A \mid R_3) \cdot P(R_3) = (1 - (1 - \alpha_1)) \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{\alpha_1 + 2}{3}$$

a.) $$P(R_1 \mid A) = \frac{P(A \cap R_1)}{P(A)} = \frac{\frac{\alpha_1}{3}}{\frac{\alpha_1 + 2}{3}} = \frac{\alpha_1}{\alpha_1 + 2}$$

b.) $$P(R_2 \mid A) = \frac{P(A \cap R_2)}{P(A)} = \frac{\frac{1}{3}}{\frac{\alpha_1 + 2}{3}} = \frac{1}{\alpha_1 + 2}$$

c.) $$P(R_3 \mid A) = \frac{P(A \cap R_3)}{P(A)} = \frac{\frac{1}{3}}{\frac{\alpha_1 + 2}{3}} = \frac{1}{\alpha_1 + 2}$$

UNIVERSITY OF TORONTO

Question 2.135

Of the travelers arriving at a small airport, 60 per cent fly on major airlines, 30 per cent fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50 per cent are traveling for business reasons, whereas 60 per cent of those arriving on private planes and 90 per cent of those arriving on other commercially owned planes are traveling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person

a) is traveling on business? $P(\text{Business}) = 0.30 + 0.18 + 0.09 = 0.57$

b) is traveling for business on a privately owned plane? $P(\text{Business} \cap \text{private}) = 0.18$

c) arrived on a privately owned plane, given that the person is traveling for business reasons? $P(\text{Private} \mid \text{Business}) = \frac{0.18}{0.57} = \frac{6}{19}$

d) is traveling on business, given that the person is flying on a commercially owned plane? $P(\text{Business} \mid \text{other}) = 0.9$

$P(\text{major}) = 0.60$ $\quad P(\text{Business} \mid \text{major}) = 0.5$ $\quad P(\text{Business} \cap \text{major}) = 0.30$

$P(\text{private}) = 0.30$ $\quad P(\text{Business} \mid \text{private}) = 0.6$ $\quad P(\text{Business} \cap \text{private}) = 0.18$

$P(\text{other}) = 0.10$ $\quad P(\text{Business} \mid \text{other}) = 0.9$ $\quad P(\text{Business} \cap \text{other}) = 0.09$