ECO227Y5 Tutorial 3

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Question: In order to verify the accuracy of their financial accounts, companies use auditors on a regular basis to verify accounting entries. The company's employees make erroneous entries 5 per cent of the time. Suppose that an auditor randomly checks three entries.

A: Find the probability distribution for Y , the number of errors detected by the auditor.

B: Construct a probability histogram for p(y).

C: Find the probability that the auditor will detect more than one error.

a.) $Y \in \{0, 1, 2, 3\}$

$_3C_0$

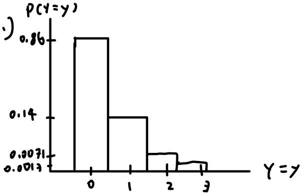
$P(Y=0) = \binom{3}{0} 0.95^3 = 0.86$

$P(Y=1) = \binom{3}{1} 0.95^2 \, 0.05 = 0.14$

$P(Y=2) = \binom{3}{2} 0.95 \; 0.05^2 = 0.0071$

$P(Y=3) = \binom{3}{3} 0.05^3 = 0.00013$

b.)



c.) $P(Y>1) = P(Y \geq 2) = 0.0071 + 0.00013 = 0.00723$

Notice this is a binomial random variable.

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Question 3.15

Question: An insurance company issues a one-year 1000 dollar policy insuring against an occurrence A that historically happens to 2 out of every 100 owners of the policy. Administrative fees are 15 dollars per policy and are not part of the company's "profit." How much should the company charge for the policy if it requires that the expected profit per policy be 50 dollars? [Hint: If C is the premium for the policy, the company's "profit" is $C - 15$ if A does not occur and $C - 15 - 1000$ if A does occur.]

Expected value $= \sum_{i=1}^{n} P(x_i) x_i$

Expected profit $= 0.02(C - 15 - 1000) + 0.98(C - 15)$

$50 = 0.02(C - 1015) + 0.98(C - 15)$

$50 = 0.02C - 20.3 + 0.98C - 14.7$

$50 = C - 35$

$C = 85$

Question 3.19

Question: Who is the king of late night TV? An Internet survey estimates that, when given a choice between David Letterman and Jay Leno, 52 per cent of the population prefers to watch Jay Leno. Three late night TV watchers are randomly selected and asked which of the two talk show hosts they prefer.
A: Find the probability distribution for Y , the number of viewers in the sample who prefer Leno.
B: Construct a probability histogram for p(y).
C: What is the probability that exactly one of the three viewers prefers Leno?

a.) $Y \in \{0,1,2,3\}$, $p = 0.52 \Rightarrow q = 1-p = 0.48$, $n=3$

$P(Y=0) = \binom{3}{0} 0.48^3 = 0.111$

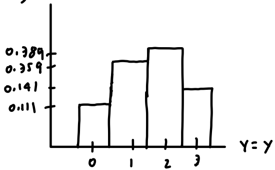
$P(Y=1) = \binom{3}{1} 0.48^2 0.52 = 0.359$

$P(Y=2) = \binom{3}{2} 0.48 \cdot 0.52^2 = 0.389$

$P(Y=3) = \binom{3}{3} 0.52^3 = 0.141$

$Y \sim Bin(3, 0.52)$

b.) $p(Y=y)$



c.) $P(Y=1) = 0.359$

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Question: Who is the king of late night TV? An Internet survey estimates that, when given a choice between David Letterman and Jay Leno, 52 per cent of the population prefers to watch Jay Leno. Three late night TV watchers are randomly selected and asked which of the two talk show hosts they prefer.
D: What are the mean and standard deviation for Y ?
E: What is the probability that the number of viewers favoring Leno falls within 2 standard deviations of the mean?

d.) $E[Y] = np = 3 \cdot 0.52 = 1.56$

$V(Y) = np(1-p) = npq = 3 \cdot 0.52 \cdot 0.48 = 0.7488$

e.) $s = \sqrt{0.7788} = 0.865$

$E(Y) \pm 2 \cdot 0.865 = [-0.17, 3.29]$

All possible values of Y fall in this interval

so $P(Y \in [-0.17, 3.29]) = 1$

Question 3.36

Question: The manufacturer of a low-calorie dairy drink wishes to compare the taste appeal of a new formula (formula B) with that of the standard formula (formula A). Each of four judges is given three glasses in random order, two containing formula A and the other containing formula B. Each judge is asked to state which glass he or she most enjoyed. Suppose that the two formulas are equally attractive. Let Y be the number of judges stating a preference for the new formula.

A: Find the probability function for Y.

B: What is the probability that at least three of the four judges state a preference for the new formula?

a.) $Y \in \{0,1,2,3,4\}$, $p = \frac{1}{3}$, $n = 4$ $\quad Y \sim Bin(4, 1/3)$

$P(Y=0) = \binom{4}{0}\left(\frac{2}{3}\right)^4 = 0.198$

$P(Y=1) = \binom{4}{1}\left(\frac{2}{3}\right)^3 \cdot \frac{1}{3} = 0.395$

$P(Y=2) = \binom{4}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = 0.296$

$P(Y=3) = \binom{4}{3}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 = 0.0988$

$P(Y=4) = \binom{4}{4}\left(\frac{1}{3}\right)^4 = 0.0123$

b.) $P(Y \geq 3) = P(Y=3) + P(Y=4)$

$= 0.0988 + 0.0123$

$= 0.1111$

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C: Find the expected value of Y.

D: Find the variance of Y.

c.) $E(Y) = np = 4 \cdot \frac{1}{3} = \frac{4}{3}$

d.) $V(Y) = np(1-p) = 4 \cdot \frac{1}{3} \cdot \frac{2}{3} = 0.\overline{8}$

Question 3.38

Question:
A: A meteorologist in Denver recorded Y = the number of days of rain during a 30-day period. Does Y have a binomial distribution? If so, are the values of both n and p given?
B: A market research firm has hired operators who conduct telephone surveys. A computer is used to randomly dial a telephone number, and the operator asks the answering person whether she has time to answer some questions. Let Y = the number of calls made until the first person replies that she is willing to answer the questions. Is this a binomial experiment? Explain.

a.) No! Binomial requires independence between trials, we cannot assume that the trials are independent.

It either rains or

b.) No, there is no fixed n, this is actually geometric.

Question 3.41

Question: A multiple-choice examination has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students who takes the examination answers each of the questions with an independent random guess. What is the probability that he answers at least ten questions correctly?

$Y \sim Bin(15, 0.2)$

$P(Y \geq 10) = P(Y=10) + P(Y=11) + P(Y=12) + P(Y=13) + P(Y=14) + P(Y=15)$

$= \sum_{i=10}^{15} \binom{15}{i} 0.2^i \, 0.8^{15-i} = 0.000132$

Question 3.45

Question: A fire-detection device utilizes three temperature-sensitive cells acting independently of each other in such a manner that any one or more may activate the alarm. Each cell possesses a probability of $p = .8$ of activating the alarm when the temperature reaches 100° Celsius or more. Let Y equal the number of cells activating the alarm when the temperature reaches 100°.
A: Find the probability distribution for Y .
B: Find the probability that the alarm will function when the temperature reaches 100°.

a.) $Y \sim Bin(3, 0.8)$

$P(Y=0) = \binom{3}{0} 0.2^3 = 0.008$

$P(Y=1) = \binom{3}{1} 0.8 \cdot 0.2^2 = 0.096$

$P(Y=2) = \binom{3}{2} 0.8^2 \cdot 0.2 = 0.384$

$P(Y=3) = \binom{3}{3} 0.8^3 = 0.512$

b.) $P(Y \geq 1) = 1 - P(Y<1) = 1 - P(Y=0)$

$= 1 - 0.008 = 0.992$

Question 3.54

Question: Suppose that Y is a binomial random variable based on n trials with success probability p and consider $Y^* = n - Y$.
A: Argue that for
$y = 0, 1, \ldots, n \quad P(Y^* = y^*) = P(n - Y = y^*) = P(Y = n - y^*)$.
B: Use the result from part A to show that
$P(Y^* = y^*) = \binom{n}{n-y^*} p^{n-y^*} q^{y^*} = \binom{n}{y^*} q^{y^*} p^{n-y^*}$.
C: The result in part B implies that Y has a binomial distribution based on n trials and "success" probability $p^* = q = 1 - p$. Why is this result "obvious"?

$P(Y^* = y^*) = P(n - Y = y^*)$
clearly

a.) Let $Y \sim Bin(n, p)$. Consider $Y^* = n - Y$, notice this is equivalent to the case where $p = 1 - p$.
You are now counting the number of failures instead of success. $Y + Y^* = n$. so $Y^* \sim Bin(n, 1-p)$

Note: $\binom{n}{y} = \frac{n!}{y!(n-y)!} = \frac{n!}{(n-y)!\,y!} = \binom{n}{n-y}$

$P(Y^* = y^*) = P(n - Y = y^*) = P(Y = n - y^*)$ clearly from the definition of $Y^* = n - Y$

b.) $P(Y^* = y^*) = \binom{n}{y^*} (1-p)^{y^*} p^{n-y^*} = \binom{n}{n-y^*} (1-p)^{y^*} p^{n-y^*}$ as needed

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$P(Y^* = y^*) = \binom{n}{n-y^*} p^{n-y^*} q^{y^*} = \binom{n}{y^*} q^{y^*} p^{n-y^*}$.

C: The result in part B implies that Y has a binomial distribution based on n trials and "success" probability $p^* = q = 1 - p$. Why is this result "obvious"?

C.) As $n - y = y^*$ is the number of unsuccessful trials which have probability $q = 1 - p$